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ON THE EFFECT OF DIFFERENT CO-ORDINATE FUNCTIONS WHEN EMPLOYING THE RAYLEIGH–RITZ METHOD IN THE CASE OF A VIBRATING RECTANGULAR PLATE WITH A FREE EDGE

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1. INTRODUCTION

When dealing with the classical, Lagrange–Germaine theory of vibrating rectangular plates, one encounters severe mathematical difficulties in the case of free edges, since satisfying the governing, natural boundary conditions is a rather difficult task. The situation is more critical if the plate possesses variable thickness, or the plate material is orthotropic or anisotropic, etc. The subject matter has been extensively discussed by A. W. Leissa in a very thorough and accurate treatment of vibrating rectangular plates [1].

In that paper, Leissa used the approximation

$$W(x, y) \simeq W_a(x, y) = \sum_{p,q} A_{p,q} X_p(x) Y_q(y), \tag{1}$$

where X_p and Y_q are normalized eigenfunctions exactly satisfying the equation of motion of a freely vibrating, uniform beam. Furthermore, X_p and Y_q satisfy the clamped, simply supported or free edge condition at the ends of the beam. Hence, clamped and simply supported plate edge conditions are exactly satisfied by use of the beam functions, but free edge conditions are only approximated.

The present study was motivated by a research program dealing with: (1) the use of Rayleigh's optimization concept [2] when trigonometric functions are employed; (2) the analysis of the relative accuracy of frequency coefficients when the classical Rayleigh–Ritz method and different co-ordinate functions are used. In the latter, the results were compared with extremely accurate results (and also upper bounds) determined by means of the finite element method [3].

The configuration shown in Figure 1 is considered and the numerical experiments reported herein are restricted to the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho h/D}\omega_1 a^2$.

2. USE OF A TRIGONOMETRIC EXPANSION CONTAINING OPTIMIZATION PARAMETERS, γ_N.

Following reference [4], it was decided to approximate the fundamental mode of vibration using a simple polynomial in y which satisfies the boundary conditions at $y = 0,b$

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(see Figure 1) and a pseudo-Fourier expansion which complies with the essential boundary conditions at $x = 0$. Consequently, one expresses the displacement amplitude as

$$W(x, y) \simeq W_a(x, y) = [(y/b)^2 - \tfrac{5}{3}(y/b)^3 + \tfrac{2}{3}(y/b)^4] \sum_{j=1}^{J} A_j \sin^2 \pi x/\gamma_j a, \qquad \gamma_1 > 1. \quad (2)$$

Substituting equation (2) in the functional

$$J[W] = (U_{\max}) - (T_{\max}), \quad (3)$$

where $U_{\max}$ is the maximum strain energy and $T_{\max}$ is the maximum kinetic energy, and minimizing it with respect to the A_j's, one obtains, from the non-triviality condition, a secular determinant the lowest root of which is the fundamental frequency coefficient Ω_1. Since

$$\Omega_1 = \Omega_1(\gamma_1, \gamma_2, \ldots, \gamma_J), \quad (4)$$

from the minimization condition

$$\partial\Omega_1/\partial\gamma_j = 0 \qquad (j = 1, 2, \ldots J) \quad (5)$$

one determines an optimized value of Ω_1. The procedure is essentially a non-linear optimization method.

3. NUMERICAL RESULTS

Numerical determinations have been performed taking μ (the Poisson ratio) to be equal to 0·30 and making $J = 1$, 2 and 3 in equation (2). In Table 1 is presented a comparison of values of Ω_1 for $a/b = 2{\cdot}5$, 1·5, 1, 2/3 and 0·4. The first line of the table depicts values determined in reference [1], evaluating the eigenvalues of a 36th order determinant, and the second, third and fourth lines depict values of Ω_1 determined by means of Rayleigh's optimization criterion and the pseudo-Fourier expansion (2). Finally, the fifth line presents fundamental eigenvalues determined by means of the finite element method, based on the very accurate element developed by Bogner *et al*. [3]. The number of nodes and degrees of freedom is indicated in the table for each case. The discretization of the structure when $a/b = 1$ is shown in Figure 2.

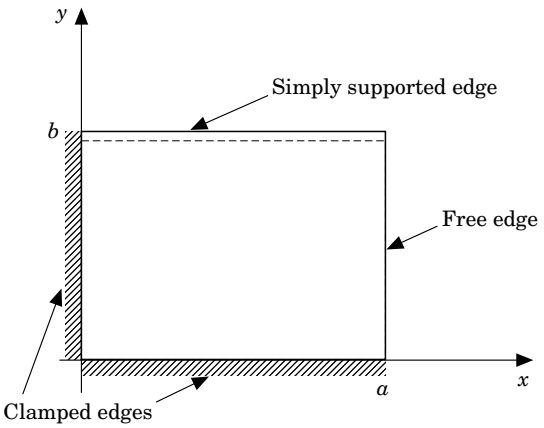

Figure 1. A rectangular plate with a free edge executing transverse vibrations.

TABLE 1

A comparison of values of $\Omega_1 = \sqrt{\rho h/D}a^2\omega_1$.

	$a/b = 2{\cdot}5$	$a/b = 1{\cdot}5$	$a/b = 1$	$a/b = 2/3$	$a/b = 0{\cdot}4$
Leissa [1]	98·100	36·6457	17·615	9·3489	5·3725
Optimization	98·8163	37·1491	18·0084	9·6405	5·5583
approach	$\gamma_1 = (2{\cdot}21)$	(2·40)	(2·64)	(3·13)	(3·28)
(trigonometric series expansion)	98·1787	36·6535	17·6231	9·3632	5·3948
	$\gamma_j = (60, 1{\cdot}04)$	(50, 1·11)	(4·22, 1·09)	(3·66, 1·09)	(3·48, 1·08)
	98·0656	36·6028	17·5935	9·3462	5·3856
	$\gamma_j = (60, 1{\cdot}00, 0{\cdot}50)$	(50, 1·08, 0·53)	(4·25, 1·07, 0·54)	(3·27, 1·00, 0·52)	(3·48, 1·06, 0·54)
F. E. method	97·7340	36·4873	17·5405	9·3184	5·3646
Degrees of freedom	448	468	484	468	448
Nodes	16 × 9	13 × 9	11 × 11	9 × 13	7 × 16

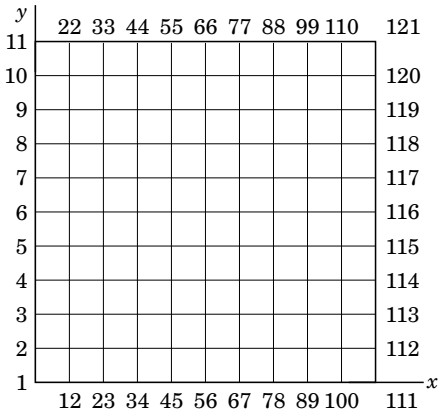

Figure 2. The discretization of the plate ($a/b = 1$).

The three-term optimized trigonometric expansion yields results which are slightly lower than the results determined in reference [1]. An exception is made for the configuration $a/b = 0{\cdot}4$. On the other hand, both sets of results are slightly higher than the very accurate results determined by means of the finite element code.

The differences are certainly not significant from a practical, designer's viewpoint but, from an academic standpoint and since the three methodologies yield upper bounds, one concludes that for the present case the ranking in accuracy is as follows: (1) the finite element results, (2) the three-term pseudo-Fourier optimized expression, and (3) the 36-term beam function solution.

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